Ch2 : Determinants

→ 2 x 2 matrix

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\det(A) = (1 \times 4) - (2 \times 3)$
$= 4 - 6 = -2$

→ 3 x 3 matrix

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ← we choose a row or a column.

$\det(A) = +1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$

$= (5 \times 9) - (6 \times 8) - 2[(4 \times 9) - (6 \times 7)] + 3[(4 \times 8) - (5$

N.B!!

If A is $n \times n$: diagonal
upper triangular
lower triangular

$\Rightarrow \det(A) = a_{11} \times a_{22} \times a_{33} \times \dots \times a_{nn}$

→ Properties of determinants

1. If the matrix A have a row of zeros or a column of zeros $\Rightarrow \det(A) = 0$
2. $\det(A) = \det(A^t)$
3. $\det(A \times B) = \det(A) \times \det(B)$

→ Finding inverse using determinant

1. Check if A is invertible

→ $\det(A) \neq 0 \Rightarrow$ invertible

→ $\det(A) = 0 \Rightarrow$ not invertible

2. Find the cofactor matrix

3. Find the adjoint matrix $[\mathrm{cof}(A)]^T$

4. Find the inverse using the formula:

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$